Riemann Sums – the basics

The German mathematician Riemann generalized the concept of sums, integrals, and how to find the area under a curve. His definition is:

f is defined on the closed interval [a, b]
Δx_i is the length of the i^{th} subinterval on [a, b]
c_i is any point in the i^{th} subinterval

Riemann Sum of f on the partition Δx:

$$\sum_{i=1}^{n} f\left(c_i\right)\Delta x_i \quad \text{where} \quad x_{i-1} \le c_i \le x_i$$

Notice that we are still adding up the areas of rectangles. Δx is the width of each rectangle and f (c_i) is the height. What has changed is that we can use any point to determine the height of the rectangle, and the width of each rectangle does not have to be same. I am sure you can see where this will eventually head - we will take the limit of this sum to find the exact area in the closed interval.

f is defined and continuous on the closed interval [a, b]
Δx is the length of the subinterval on [a, b]
c_k is any point in the i^{th} subinterval

The Integral of f:

$$\lim_{n\to\infty}\sum_{k=1}^{\infty} f\left(c_k\right)\Delta x = \int_a^b f(x)\,dx$$

When the widths, the subintervals, are different for each rectangle, we define the **norm** of the partition, $\|\Delta\|$, as being the largest subinterval. If every subinterval is the same length, the partition is **regular**, and the length of each can be found by dividing the length of the partition by the number of subintervals.

All this complicated looking limits notation boils down to the relatively simple looking integral notation. You will have to be able to rewrite the limit notation as an integral on the homework (and on the test). And we will be using the integral notation for the rest of the module (and the next modules).